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The effect of radiation on free convection from a porous vertical plate

M. A. Hossain[a], M. A. Alim[a,‡], D. A. S. Rees[b,*]

[a] *Department of Mathematics, University of Dhaka, Dhaka 1000, Bangladesh*
[b] *Department of Mechanical Engineering, University of Bath, Bath BA2 7AY, U.K.*

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Abstract

We determine the effect of radiation on the natural convection flow of an optically dense incompressible fluid along a uniformly heated vertical plate with a uniform suction. The governing nonsimilar boundary-layer equations are analyzed using (i) a series solution for small values of ξ (a scaled streamwise coordinate); (ii) an asymptotic solution for large ξ; and (iii) a full numerical solution. The solutions are expressed in terms of the local shear stress and local rate of heat transfer. The effects of varying the Prandtl number, Pr, the radiation parameter, R_d, and the surface temperature parameter, θ_w, are determined. © 1998 Elsevier Science Ltd. All rights reserved.

Nomenclature

a Rosseland mean absorption coefficient
C_p specific heat at constant pressure
f dimensionless streamfunction
F dimensionless streamfunction in asymptotic analysis
g acceleration due to gravity
Q_w rate of heat transfer
Pr Prandtl number
R_d Planck number (radiation–conduction parameter)
T temperature of the fluid
T_w temperature of the heated surface
T_∞ temperature of the ambient fluid
u velocity in the x-direction
v velocity in the y-direction
V wall suction velocity
x streamwise coordinate measuring distance along the surface
y cross-stream coordinate measuring distance normal to the surface.

Greek symbols

α equal to $4/3R_d$
β coefficient of cubical expansion
Δ equal to $\theta_w - 1$
ΔT equal to $T_w - T_\infty$
ζ similarity variable
η similarity variable
θ dimensionless temperature function
θ_w surface temperature ratio to the ambient fluid
κ coefficient of thermal diffusivity
μ the dynamic viscosity
ν the kinematic viscosity
ξ a scaled streamwise coordinate
ρ density of the fluid
σ Stefan–Boltzmann constant
σ_s scattering coefficient
τ coefficient of skin friction.

1. Introduction

Radiation effects on free convection flow are important in the context of space technology and processes involving high temperatures, and very little is known about the effects of radiation on the boundary-layer flow of a radiating fluid past a body. The inclusion of radiation effects in the energy equation, however, leads to a highly nonlinear partial differential equation. Soundalgekar and

* Corresponding author. Tel.: 00 44 1225 826775; fax: 00 44 1225 826928; e-mail: ensdasr@bath.ac.uk

† Present address: Department of Mathematics, Bangladesh University of Engineering and Technology, Dhaka 1000, Bangladesh.

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Takhar [1] have studied radiation effects on free convection flow of a gas past a semi-infinite flat plate using the Cogley–Vincenti–Giles equilibrium model (Cogley et al. [2]). Very recently, Hossain and Takhar [3] have analyzed the effect of radiation using the Rosseland diffusion approximation which leads to nonsimilar solutions for the forced and free convection flow of an optically dense viscous incompressible fluid past a heated vertical plate with uniform free stream velocity and surface temperature. In this analysis consideration had been given to grey gases that emit and absorb but do not scatter thermal radiation. We note that the Rosseland diffusion approximation provides one of the most straightforward simplifications of the full integro/partial differential equations governing such flows. Limitations of this approximation are discussed briefly in Özişik [24]. Using a suitable set of transformations, the boundary-layer equations governing the flow in [3] were reduced to a locally nonsimilar form from which may be recovered both the forced convection and free convection limits. The resulting nonsimilar equations were solved using an implicit finite difference method.

Convective boundary-layer flows are often controlled by injecting or withdrawing fluid through a porous bounding heated surface. This can lead to enhanced heating or cooling of the system and can help to delay the transition from laminar to turbulent flow. Previous work on the effects of blowing and suction on free convection boundary-layers without radiative effects have been confined to cases with a prescribed wall temperature. Eichhorn [4], for example, obtained those power-law variations in surface temperature and transpiration velocity which give rise to a similarity solution for the flow from a vertical surface.

The case of uniform suction and blowing through an isothermal vertical wall was treated first by Sparrow and Cess [5]; they obtained a series solution which is valid near the leading edge. This problem was considered in more detail by Merkin [6], who obtained asymptotic solutions, valid at large distances from the leading edge, for both the suction and blowing. Using the method of matched asymptotic expansions, the next order corrections to the boundary-layer solution for this problem were obtained by Clarke [7], who extended the range of applicability of the analyses by not invoking the usual Boussinesq approximation. The effect of strong suction and blowing from general body shapes which admit a similarity solution has been given by Merkin [8]. A transformation of the equations for general blowing and wall temperature variations has been given by Vedhanayagam et al. [9]. The case of a heated isothermal horizontal surface with transpiration has been discussed in some detail first by Clarke and Riley [10, 11], and then more recently by Lin and Yu [12]. In the present paper we determine the effect of radiation on natural convection flow of an optically thick viscous incompressible flow past a heated vertical porous plate with a uniform surface temperature and a uniform rate of suction where radiation is included by assuming the Rosseland diffusion approximation. Asymptotic solutions are obtained both near to and far from the leading edge and numerical solutions at intermediate locations are obtained using the Keller-box method (Keller [13]). It is found that the presence of suction serves to thin the boundary-layer at large distances from the leading edge when compared with the analogous solution for an impermeable heated surface. Radiation effects are found not to modify this qualitative behaviour, but they do affect the quantitative results.

2. Mathematical formulation

Consider a semi-finite porous plate at a uniform temperature T_w which is played vertical in a quiescent fluid of infinite extent at constant temperature T_∞. The fluid is assumed to be a grey, emitting and absorbing, but non-scattering medium. In the present paper the following assumptions are made: (i) variations in fluid properties are limited only to those density variations which affect the buoyancy terms, (ii) viscous dissipation effects are negligible, and (iii) the radiative heat flux in the x-direction is considered negligible in comparison with that in the y-direction, where the physical coordinates (x, y) are chosen such that x is measured from the leading edge in the streamwise direction and y is measured normal to the surface of the plate. The coordinates system and the flow configuration are shown in Fig. 1.

Under the usual Boussinesq approximation, the con-

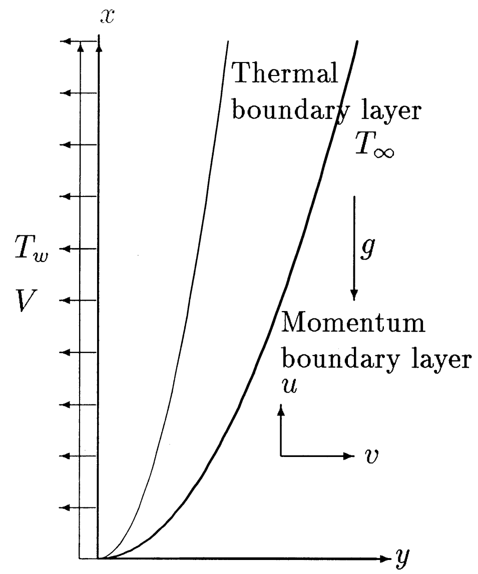

Fig. 1. The coordinate system and the physical model.

servation equations for the steady, laminar, two-dimensional boundary-layer flow problem under consideration can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}+\rho g\beta(T-T_\infty) \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\kappa\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y} \tag{3}$$

where, u and v are the velocity components in the x and y, directions, respectively, v is the coefficient of viscosity, ρ is the density of ambient fluid, g is the acceleration due to gravity, β is the coefficient of cubical expansion, κ is the coefficient of thermal diffusivity and T is the temperature of the fluid and q_r is the component of radiative flux. The quantity q_r on the right-hand side of equation (3) represents the radiative heat flux in the y direction. In order to reduce the complexity of the problem and to provide a means of comparison with future studies which will employ a more detailed representation for the radiative heat flux, the optically thick radiation limit is considered in the present analysis. Thus, the radiative heat flux term is simplified by using the Rosseland diffusion approximation (Sparrow and Cess [5]) for an optically thick fluid according to,

$$q_r=-\frac{16\sigma T^3}{3\alpha_R}\frac{\partial T}{\partial y} \tag{4}$$

where σ is the Stefan–Boltzmann constant, α_R is the Rosseland mean absorption coefficient. This approximation is valid at points optically far from the bounding surface, and is good only for intensive absorption, that is, for an optically thick boundary-layer. Despite these shortcomings, the Rosseland approximation has been used with success in a variety of problems ranging from the transport of radiation through gases at low density to the study of the effects of radiation on blast waves by nuclear explosion (Ali et al. [15]).

The boundary conditions for the present problem are:

$$\begin{aligned}&x=0,\quad y>0:\quad u=0,\quad T=T_\infty\\ &y=0,\quad x>0:\quad u=0,\quad v=-V,\quad T=T_w\\ &y\to\infty,\quad x>0:\quad u=0,\quad T=T_\infty.\end{aligned} \tag{5}$$

In equation (5) V represents the suction velocity of fluid through the surface of the plate. In this paper we shall consider only the suction case (rather than blowing) and, therefore, V is taken as positive throughout.

Near the leading edge, the boundary-layer is much like that of the free convection boundary-layer in the absence of suction, although much further downstream suction it will be found to dominate the flow. Therefore, the following group of transformations are introduced:

$$\begin{aligned}&\eta=\frac{Vy}{v\xi},\quad \xi=V\left\{\frac{4x}{v^2 g\beta(T_w-T_\infty)}\right\}^{1/4}\\ &\psi=V^{-3}v^2 g\beta(T_w-T_\infty)\xi^3\{f(\xi,\eta)\pm\tfrac{1}{4}\xi\}\\ &\frac{T-T_\infty}{T_w-T_\infty}=\theta(\xi,\eta),\quad \frac{T_w}{T_\infty}=\theta_w,\quad R_d=\frac{\kappa\alpha_R}{4\sigma T_\infty^3}\end{aligned} \tag{6}$$

where ψ is the stream function satisfying equation (1) (i.e. $u=\psi_y$ and $v=-\psi_x$), θ_w is the surface temperature parameter and R_d is the radiation parameter. In equation (6) the plus sign corresponds to suction and the minus sign for blowing or injection. In this paper we restrict attention to suction only.

It should be mentioned that, the optically thick approximation should be valid for relatively low values of conduction–radiation parameter, R_d. According to Ali et al. [15], some gases with their R_d-values are: $R_d=10$–30: carbon dioxide (100–650°F) with corresponding Prandtl number range 0.76–0.6, ammonia vapour (120–400°F) with corresponding Prandtl number range 0.88–0.84, $R_d=30$–200: water vapour (220–900°F) with corresponding Prandtl number $Pr=1$ the R_d values lies between 30–200.

Equations (2) and (3) together with (4) thus become

$$f'''+3ff''-2f'^2+\theta+\xi f''=\xi\left\{f'\frac{\partial f'}{\partial\xi}-f''\frac{\partial f}{\partial\xi}\right\} \tag{7}$$

$$\frac{1}{Pr}\left[\left\{1+\frac{4}{3R_d}(1+(\theta_w-1)\theta)^3\right\}\theta'\right]'+3f\theta'+\xi\theta'=\xi\left(f'\frac{\partial\theta}{\partial\xi}-\theta'\frac{\partial f}{\partial\xi}\right) \tag{8}$$

and the corresponding boundary conditions transform to

$$\begin{aligned}&f=0,\quad f'=0,\quad \theta=1\quad \text{at } \eta=0\\ &f'=0,\quad \theta=0\quad \text{as } \eta\to\infty\end{aligned} \tag{9}$$

where $Pr=v/\kappa$ is the Prandtl number. Equations (7)–(8) are the locally nonsimilar equations governing the flow under consideration.

The solutions of equations (7)–(8) enable us to calculate the nondimensional velocity components, $\bar{u}$, and $\bar{v}$, from the following expressions:

$$\bar{u}=\left(\frac{V}{g\beta(T_w-T_\infty)}\right)u=\xi^2 f'(\xi,\eta) \tag{10}$$

$$\bar{v} = v/V$$
$$= \xi^{-1}\left\{3f + \xi - \eta f' + \xi \frac{\partial f}{\partial \xi}\right\} \tag{11}$$

and the shear stress τ_w, and the rate of heat transfer Q_w, at the surface of the plate from

$$\tau_w = \left(\frac{V}{g\beta\Delta T}\right)\left(\frac{\partial u}{\partial y}\right)_{y=0}$$
$$= \xi f''(\xi, 0) \tag{12}$$
$$Q_w = -\left(\frac{v}{V\Delta T}\right)\left(\frac{\partial T}{\partial y}\right)_{y=0}$$
$$= \left(1 + \frac{4}{3R_d}\theta_w^3\right)\xi^{-1}\theta'(\xi, 0) \tag{13}$$

where $\Delta T = T_w - T_\infty$. The set of partial differential equations (7) and (8) governing the flow could be integrated by reducing to sets of nonlinear ordinary differential equations by the local similarity method (Sparrow and Yu [16]; Chen [17] Hossain et al. [18]). Generally, the convergence of this solution method becomes increasingly difficult for large values of ξ. It is important to investigate in detail the flow at both large and small values of ξ. At intermediate values of ξ we employ in Keller-box method, a scheme designed especially for parabolic partial differential equations (Cebeci and Keller [19]). In this method, the present system of equations are reduced to a system consisting of equations which are first order in η. Central difference approximations based halfway along the both η and ξ intervals are made and the resulting set of nonlinear difference equations are solved by using the Newton–Raphson quasi-linearization method. The Jacobian matrix has a block-tridiagonal structure and the difference equations are solved using a block-matrix version of the Thomas algorithm; the details of the computational procedure have been discussed further by Hossain et al. [18, 20]. A wide range of numerical results have been obtained using this method, but we present here just a small selection. Finally, the computed solutions obtained above are validated by a series expansion method for small ξ and by an asymptotic analysis for large values of ξ.

It should be noted that, in the absence of radiative effects ($1/R_d = 0$), the present problem (7)–(8) has been investigated by Merkin [8] employing the method of series solutions and asymptotic solutions, respectively, for small ξ and large ξ.

3. Series solution for small-ξ (SRS)

The series solution is valid for sufficiently small values of ξ, that is, either for sufficiently small distances from the leading edge or for small values of V—see the definition of ξ in (5). Accordingly, the functions $f(\xi, \eta)$ and $\theta(\xi, \eta)$ are expanded in a power series in ξ, that is, we take

$$f(\xi, \eta) = \sum_{n=0}^{\infty} \xi^n f_n(\eta) \quad \text{and} \quad \theta(\xi, \eta) = \sum_{n=0}^{\infty} \xi^n \theta_n(\eta). \tag{14}$$

Substituting the above expansion into equations (7)–(8) and equating the coefficients of various powers of ξ, we get the following equations:

$$f'''_0 + 3f_0 f''_0 - 2f'^2_0 + \theta_0 = 0 \tag{15}$$

$$[1 + \alpha(1+\Delta\theta_0)^3]\theta''_0 + 3\alpha\Delta(1+\Delta\theta_0)^2\theta'^2_0 + 3\,Pr\, f_0\theta'_0 = 0 \tag{16}$$

where $\alpha = (4/3R_d)$ and $\Delta = \theta_w - 1$,

$$f'''_1 + 3f''_1 f_0 + 4f_1 f''_0 - 5f'_1 f'_0 + \theta_1 + f''_0 = 0 \tag{17}$$

$$[1 + \alpha(1+\Delta\theta_0)^3]\theta''_1 + 3\alpha\Delta(1+\Delta\theta_0)^2(\theta''_0\theta_1 + 2\theta'_1\theta'_0)$$
$$+ 6\alpha(\Delta\theta'_0)^2\theta_1(1+\Delta\theta_0) + Pr[3(f_0\theta'_1 + f_1\theta'_0)$$
$$+ \theta'_0 - f'_0\theta_1 + f_1\theta'_0] = 0 \tag{18}$$

$$f'''_2 + 3(f''_2 f_0 + f''_1 f_1 + f''_0 f_2) - 2(2f'_2 f'_0 + f'^2_1) + \theta_2 + f''_1$$
$$= 2(f'_2 f'_0 - f_2 f''_0) + (f'_1 f'_1 - f_1 f''_1) \tag{19}$$

$$[1 + \alpha(1+\Delta\theta_0)^3]\theta''_2 + 3\alpha\Delta\theta''_1\theta_1(1+\Delta\theta_0)^2$$
$$+ 3\alpha\Delta\theta''_0[\theta_2 + \Delta(2\theta_2\theta_0 + \theta_1^2) + \Delta^2\theta_0(\theta_2\theta_0 + \theta_1^2)]$$
$$+ 3\alpha\Delta[(2\theta'_2\theta'_0 + \theta'^2_1)(1+\Delta\theta_0)^2$$
$$+ 4\Delta\theta'_1\theta_1\theta_0(1+\Delta\theta_0) + \Delta\theta'^2_0\{2\theta_2 + \Delta(2\theta_2\theta_0 + \theta_1^2)\}]$$
$$+ 3Pr(\theta'_2 f_0 + \theta'_1 f_1 + \theta'_0 f_2) + Pr\,\theta'_1 \tag{20}$$
$$= Pr[2(\theta_2 f'_0 - \theta'_0 f_2) + (\theta_1 f'_1 - \theta'_1 f_1)].$$

The corresponding boundary conditions are

$$f_i(0) = f'_i(0) = \theta_i(0) = 0, \quad \theta_0(0) = 1$$
$$f'_i(\infty) = \theta_i(\infty) = 0 \tag{21}$$

for $i = 0, 1, 2$. The coupled equations (15) and (16) are nonlinear, whereas (17)–(21) are linear, and these may be solved independently pairwise one after another. The implicit Runge–Kutta–Butcher (Butcher [21]) initial value solver together with the Nachtsheim–Swigert iteration scheme of [22] is employed to solve the first pair (15) and (16) and the other pairs up to $O(\xi^2)$ are solved using the method of superposition (Na [23]). The resulting solutions are expressed in terms of local skin-friction, τ_w, and the rate of heat transfer, Q and compared with the solutions obtained from the finite difference method for which the coefficient of the series has been taken up to $O(\xi^2)$.

4. Asymptotic solution for large-ξ (ASY)

In this section attention shall be given to the behaviour of the solution to equations (7) and (8) when ξ is large. Given that we consider only the suction case, for which we take the positively signed terms in (7)–(8), an order-of-magnitude analysis of the various terms in these equations shows that the largest are $\xi f''$ in (7) and $\xi\theta'$ in (8). In their respective equations, both terms have to be balanced and the only way to do this is to assume that η is small, and hence η-derivatives are large.

Given that $\theta = O(1)$ as $\xi \to \infty$, it is necessary to find the appropriate scalings for f and η. On balancing the f''', θ, and $\xi f''$ terms in (7), it is found that $\eta = O(\xi^{-1})$ and $f = O(\xi^{-3})$ are $\xi \to \infty$. Therefore, the following substitutions are made,

$$f = \xi^{-3}F(\xi,\zeta), \quad \theta = \theta(\xi,\zeta) \quad \text{and} \quad \eta = \zeta/\xi. \tag{22}$$

Equations (7) and (8) together with (22) become

$$F''' + F'' + \theta = \frac{1}{\xi^3}\left\{F'\frac{\partial F'}{\partial \xi} - F''\frac{\partial F}{\partial \xi}\right\} \tag{23}$$

$$\frac{1}{Pr}\left[\left\{1+\frac{4}{3R_d}(1+(\theta_w-1)\theta)^3\right\}\theta'\right]' + \theta' = \frac{1}{\xi^3}\left(F'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial F}{\partial \xi}\right) \tag{24}$$

where primes denote differentiation with respect to ζ. This convention will be used for the rest of this section. The boundary conditions are simply

$$F = 0, \quad F' = 0, \quad \theta = 1 \quad \text{at } \zeta = 0$$
$$F' = 0, \quad \theta = 0 \quad \text{as } \zeta \to \infty. \tag{25}$$

Equations (23) and (24) may be solved in terms of an inverse power of series in ξ.

The solution is valid for sufficiently large values of ξ. Accordingly, the functions $F(\xi,\zeta)$ and $\theta(\xi,\zeta)$ are expanded in a power series in negative powers of ξ, that is, we take

$$F(\xi,\zeta) = \sum_{n=0}^{\infty}\xi^{-n}F_n(\zeta) \quad \text{and} \quad \theta(\xi,\zeta) = \sum_{n=0}^{\infty}\xi^{-n}\theta_n(\zeta). \tag{26}$$

Substituting the above expansion into equations (23) and (24) and equating the coefficients of various powers of ξ, we get the following equations:

$$F'''_0 + F''_0 + \theta_0 = 0 \tag{27}$$

$$[1+\alpha(1+\Delta\theta_0)^3]\theta''_0 + 3\alpha\Delta(1+\Delta\theta_0)^2\theta'^2_0 + Pr\,\theta'_0 = 0 \tag{28}$$

where we recall once more that $\alpha = (4/3R_d)$ and $\Delta = \theta_w - 1$,

$$F'''_1 + F''_1 + \theta_1 = 0$$

$$[1+\alpha(1+\Delta\theta_0)^3]\theta''_1 + 3\alpha\Delta(1+\Delta\theta_0)^2(\theta''_0\theta_1 + 2\theta'_1\theta'_0) + 6\alpha(\Delta\theta'_0)^2\theta_1(1+\Delta\theta_0) + Pr\,\theta'_1 = 0 \tag{30}$$

$$F'''_2 + F''_2 + \theta_2 = 0 \tag{31}$$

$$\begin{aligned}&[1+\alpha(1+\Delta\theta_0)^3]\theta''_2 + 3\alpha\Delta\theta''_1\theta_1(1+\Delta\theta_0)^2 \\ &+ 3\alpha\Delta\theta''_0[\theta_2 + \Delta(2\theta_2\theta_0 + \theta_1^2) + \Delta^2\theta_0(\theta_2\theta_0 + \theta_1^2)] \\ &+ 3\alpha[gkdE][2\theta'_2\theta'_0 + \theta'^2_1)(1+\Delta\theta_0)^2 \\ &+ 4\Delta\theta'_1\theta_1\theta_0(1+\Delta\theta_0) + \Delta\theta_0^2\{2\theta_2 + \Delta(2\theta_2\theta_0 + \theta_1^2)\}] \\ &+ Pr\,\theta'_2 = 0.\end{aligned} \tag{32}$$

The corresponding boundary conditions are

$$F_i(0) = F'_i(0) = \theta_i(0) = 0, \quad \theta_0(0) = 1$$
$$F'_i(\infty) = \theta_i(\infty) = 0 \tag{33}$$

for $i = 0, 1, 2$. From these solutions the local skin friction, τ_w, and rate of heat transfer, Q_w, are found to be

$$\tau_w = F''(\xi, 0) \tag{34}$$

and

$$Q_w = \left(1 + \frac{4}{3R_d}\theta_w^3\right)\theta'(\xi,0) \tag{35}$$

where these expressions are valid for all values of Pr.

For $\alpha = 0$ as $R_d \to \infty$, solutions of the equations (27)–(32) lead to the following form of the skin-friction, τ_w, and the rate of heat transfer, Q_w:

$$\tau_w \approx \frac{1}{Pr} + O(\xi^{-4}) \tag{36}$$

and

$$Q_w \approx Pr + O(\xi^{-6}). \tag{37}$$

5. Results and discussion

The results obtained by three distinct methods namely, a series solution method for small values of ξ, the Keller-box method for $\xi \in [0,\infty)$ and an asymptotic method for large ξ, have been employed to integrate the coupled partial differential equations (7) and (8) which describe the natural convection flow along a uniformly heated vertical porous plate with the effect of radiation.

The results are obtained in terms of the local skin-friction, τ_w, and the local rate of heat transfer, Q_w for different values of the aforementioned physical parameters and these are shown in tabular form in Table 1 and graphically in Figs 2–5.

The numerical results of τ_w and Q_w obtained by the series solution and the asymptotic solution for different values of θ_w are compared with those obtained by the finite difference solutions in Table 1 in the range of $\xi \in [0, 12]$ for $Pr = 1$ and $R_d = 20$ and for values of the surface temperature parameters of 1.1, 1.5 and 3.0. From this table it can be seen that the results for the series solution method as well as the asymptotic method are in good agreement with the Keller box solutions. It can also be noticed that the skin-friction and the rate of heat transfer both increase as the surface temperature parameter increases. For all values of the surface temperature parameter, θ_w, the heat transfer rate leads to the asymptotic value Pr as $\xi \to \infty$.

Table 1
Numerical values of skin-friction and the rate of surface heat transfer obtained by different methods for different values of θ_w while $Pr = 1.0$ and $r_d = 20.0$

	Local skin friction		Surface heat flux	
	τ_w		Q_w	
ξ	SRS and ASY	FDS	SRS and ASY	FDS
$\theta_w = 1.1$				
0.10	0.0655†	0.0655	6.4133†	6.4627
0.20	0.1320†	0.1316	3.4352†	3.4928
0.40	0.2671†	0.2647	1.9553†	2.0229
0.60	0.4041†	0.3963	1.4701†	1.5439
0.80	0.5416†	0.5235	1.2335†	1.3247
1.00	0.6783†	0.6429	1.0964†	1.1995
1.50	1.0787‡	0.8874	1.0000‡	1.0574
2.00	1.0787‡	1.0278	1.0000‡	1.0120
3.00	1.0787‡	1.0769	1.0000‡	1.0001
4.00	1.0787‡	1.0771	1.0000‡	1.0001
5.00	1.0787‡	1.0772	1.0000‡	1.0001
$\theta_w = 1.5$				
0.10	0.0664†	0.0665	6.7190†	6.7390
0.20	0.1340†	0.1339	3.5941†	3.6266
0.40	0.2717†	0.2699	2.0419†	2.0877
0.60	0.4116†	0.4051	1.5336†	1.5892
0.80	0.5526†	0.5365	1.2862†	1.3511
1.00	0.6934†	0.6608	1.1433†	1.2177
1.50	1.1358‡	0.9196	0.9999‡	1.0652
2.00	1.1358‡	1.0745	1.0000‡	1.0146
3.00	1.1358‡	1.1350	1.0000‡	1.0005
4.00	1.1358‡	1.1355	1.0000‡	1.0001
5.00	1.1358‡	1.1358	1.0000‡	1.0001
$\theta_w = 2.5$				
0.10	0.0708†	0.0709	8.1122†	8.0844
0.20	0.1432†	0.1433	4.3306†	4.2858
0.40	0.2916†	0.2917	2.4491†	2.4003
0.60	0.4447†	0.4423	1.8303†	1.7843
0.80	0.6004†	0.5922	1.5272†	1.4860
1.00	0.7578†	0.7379	1.3503†	1.1098
1.50	1.1514‡	1.0613	1.0747‡	1.1098
2.00	1.3129‡	1.2871	1.0257‡	1.0327
3.00	1.4189‡	1.4209	1.0004‡	1.0006
4.00	1.4220‡	1.4236	1.0001‡	1.0001
5.00	1.4244‡	1.4244	1.0001‡	1.0000

† For small ξ and ‡ for large ξ.

The effects of varying the radiation parameter, R_d, on both the skin-friction and the local rate of heat transfer are shown in Figs 2 and 3, respectively. In Figs 2 and 3 it can be seen that the match between the Keller-box solution and the small-ξ and large-ξ analyses is very good indeed. Changes in R_d lead to changes in the asymptotic value of the skin friction, but not in the asymptotic rate of heat transfer. In general, however, both the skin-friction and the rate of heat transfer decrease as R_d increases. In Figs 2(a) and 2(b) the curves corresponding to $1/R_d = 0$ (i.e. where the conduction–radiation effect is absent) are in excellent agreement with those of Merkin [8].

Figures 4 and 5 show how the results are affected by changing the Prandtl number with both R_d and θ_w held fixed. Again the match between the various methods of solution in their respective domains of validity is excellent. As the Prandtl increases, the skin-friction decreases, but the rate of heat transfer increases. The asymptotic limit of both these quantities is dependent on Pr—all these observations are consistent with (36) and (37), as is the fact that the approach to the asymptotic solutions is very rapid.

The velocity and temperature distributions obtained by the finite difference method for various values of the governing parameters, are displayed in Figs 6–8. The aim of these figures is to display how the profiles vary in ξ, the scaled streamwise coordinate. It is shown that, relative to constant values of η, both the velocity and the temperature decrease in magnitude as ξ increases. Thus the numerical results in Fig. 6 indicate that both the momentum and the thermal boundary-layer thickness decrease in terms of η at increasing distances from the leading edge. This provides qualitative support of the form of the asymptotic analysis where it is shown that suction effects lead to a constant thickness boundary layer in terms of y. In order that above results may be useful for experimental verification, we can determine the percentage increase in the maximum velocity in the boundary-layer. Thus when conduction–radiation effects are present in a fluid with $Pr = 1.0, \theta_w = 1.1.$ and $R_d = 10.0$, the velocity boundary-layer decreases by 17.2, 37.4 and 83.1% when ξ increases from 0.0–1.0, 2.0 and 5.0, respectively. The corresponding decreases in the thickness of the thermal boundary-layer calculated at $\eta = 1.02$ are 17.9, 35.3 and 77.8%.

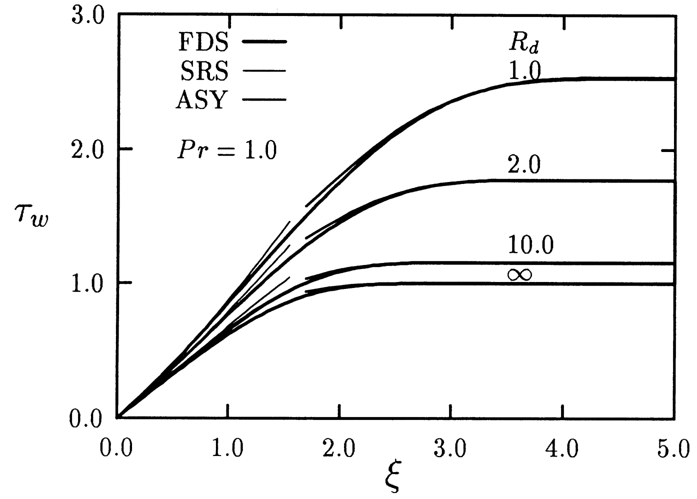

Fig. 2. Local skin-friction against ξ for different R_d with $\theta_w = 1.1$ and $Pr = 1.0$. The curves for $R_d = \infty$ represent those obtained by Merkin [8].

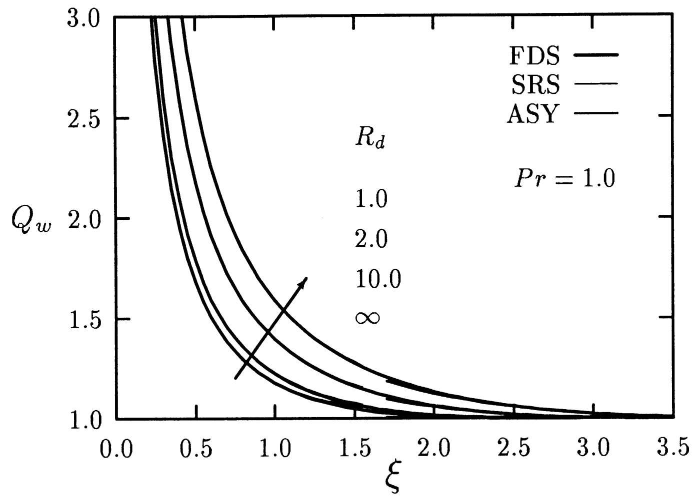

Fig. 3. Local heat transfer against ξ for different R_d with $\theta_w = 1.1$ and $Pr = 1.0$. The curves for $R_d = \infty$ represent those obtained by Merkin [8].

In Fig. 7(a) and (b) we display the effect of varying the conduction–radiation parameter, R_d, on the velocity and temperature distribution in the boundary layer. It can be seen that an increase of the conduction–radiation parameter decreases the local velocity as well as the temperature. Therefore increases in the value of R_d leads to a decrease in both the momentum and thermal boundary-layer thicknesses. From the calculated values of the velocity when $Pr = 1.0$, $\theta_w = 1.1$ and $\xi = 2.0$, the maximum velocity increases by 37.41, 52.0, 63.21, 75.7 and 79.5% with the decrease in the value of the conduction–radiation parameter, R_d from ∞–10, 2.0, 1.0, 0.5 and 0.1. The corresponding increase of the local temperature calculated at $\eta = 1.02$ are 32.7, 49.7, 63.7, 79.6 and 102.5%.

In Fig. 8(a) and (b) we see the effect of varying the surface parameter, θ_d, on the velocity and the temperature distributions. One may observe that the both the velocity and the temperature distribution and hence the momentum thickness and the thermal boundary-layer thickness increase due to increase in this parameter. As before, for $R_d = 10.0$ and $Pr = 1.0$ the value of the maximum velocity increases by 10.5, 31.1 and 59.06% as the value of the surface temperature parameter θ_w is increased from 1.1–1.5, 2.0 and 2.5. Finally, when the value of θ_w is

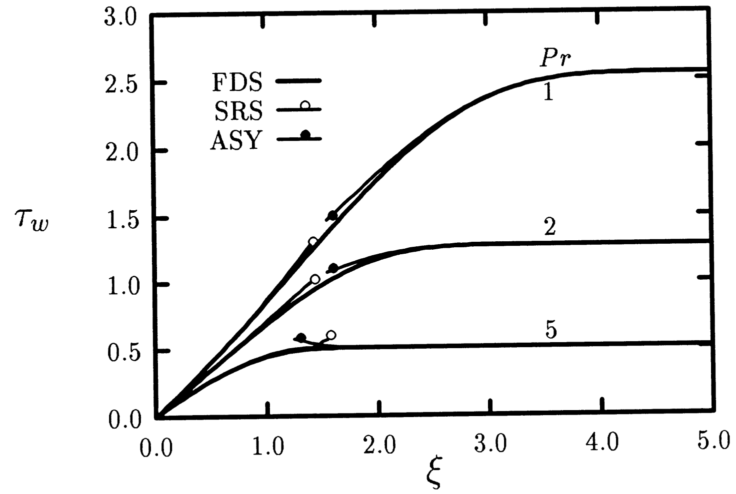

Fig. 4. Local skin-friction against ξ for different Pr for $\theta_w = 1.1$ and $R_d = 1.0$.

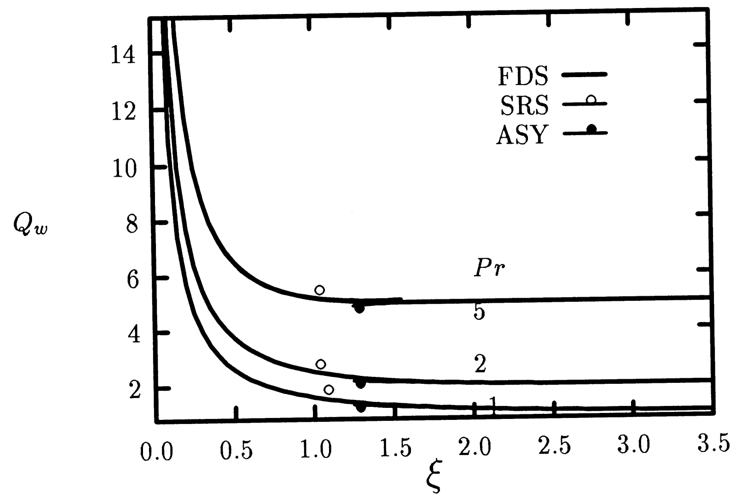

Fig. 5. Local heat transfer against ξ for different Pr for $\theta_w = 1.1$ and $R_d = 1.0$.

increase from 1.1–1.5, 2.0 and 2.5 the value of the local temperature at $\eta = 1.02$ is found to be increased by 15.3, 47.5 and 100.0%.

For the sake of brevity we have not displayed isotherm plots to indicate how the boundary-layer behaves, firstly as a function of ξ, and secondly how its thickness varies at any one chosen value of ξ for different values of the governing parameters. When the boundary-layer begins to grow at the leading edge it resembles a free convection layer and its thickness is proportional to $x^{1/4}$. Further downstream the growth in thickness is halted and it eventually becomes a constant thickness boundary-layer because of the surface suction. We also note that separate increases in the value of R_d and θ_w cause the boundary-layer to increase in relative thickness at any chosen value of ξ. However, when the boundary-layer attains the asymptotic uniform thickness, any changes brought about by variations in these parameters are negligible.

6. Conclusions

In this paper we have sought to determine how the presence of radiation effects alter the free convection boundary-layer flow from a uniformly heated surface with suction. The presence of radiation serves to introduce two extra parameters into the problem, namely, R_d and θ_w. The detailed effects of varying Pr, and R_d and θ_w

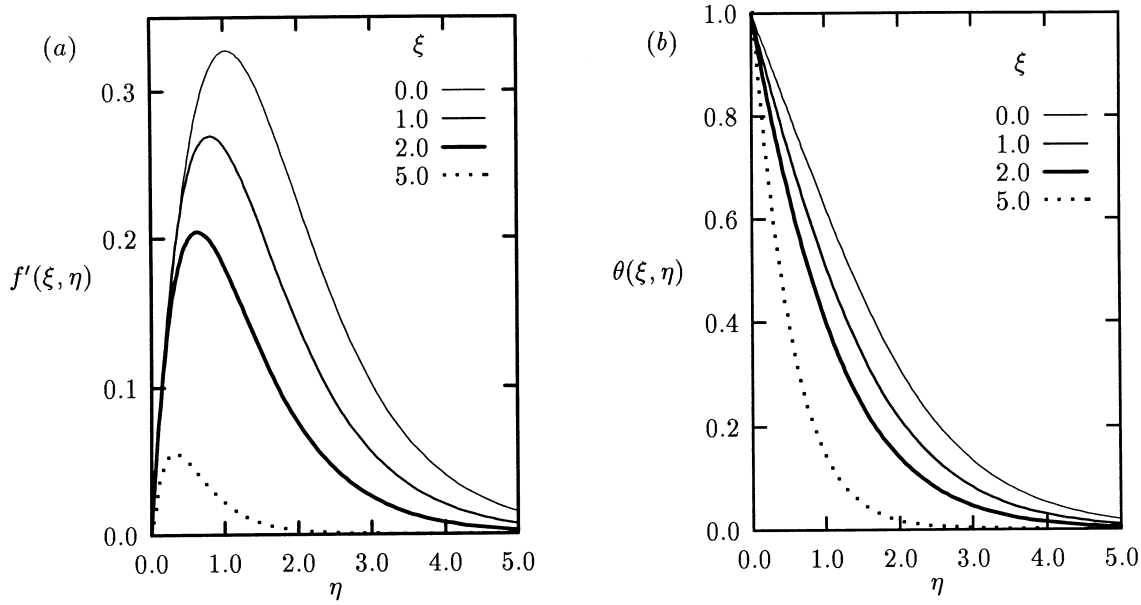

Fig. 6. (a) Velocity and (b) temperature distribution against η for different ξ for $R_d = 1.0$, $\theta_w = 1.1$ and $Pr = 1.0$.

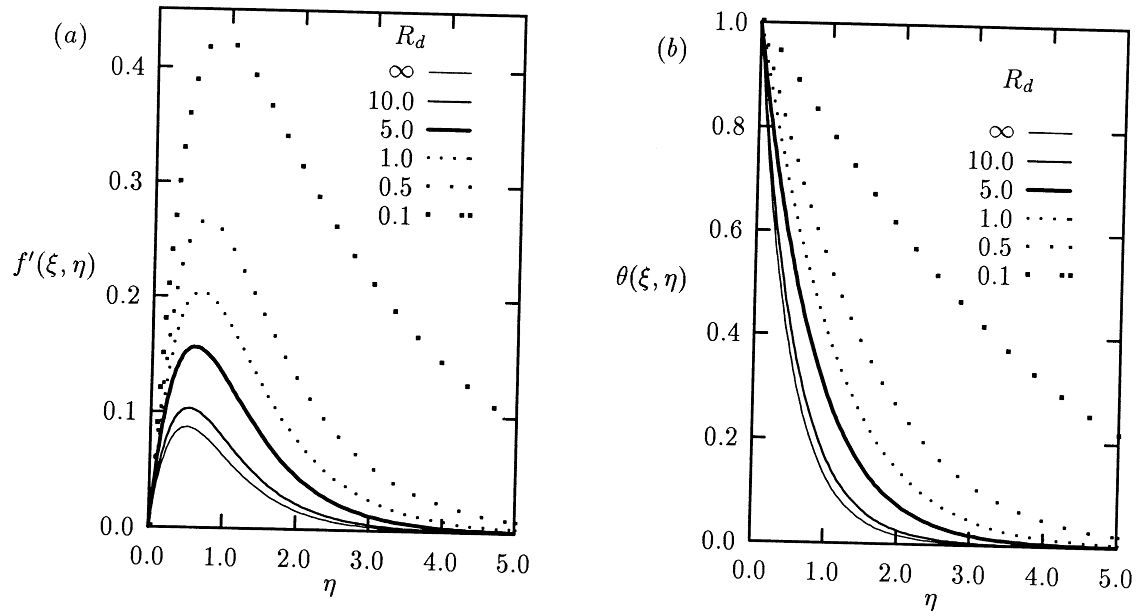

Fig. 7. (a) Velocity and (b) temperature distribution against η for different R_d at $\xi = 2$, $\theta_w = 1.1$ and $Pr = 1.0$.

are complicated and a small selection have been presented above. In general, an increase in R_d serves to thin the layer and an increase in θ_w serves to thicken the layer. The presence of suction ensures that its ultimate fate as ξ increases is a layer of constant thickness.

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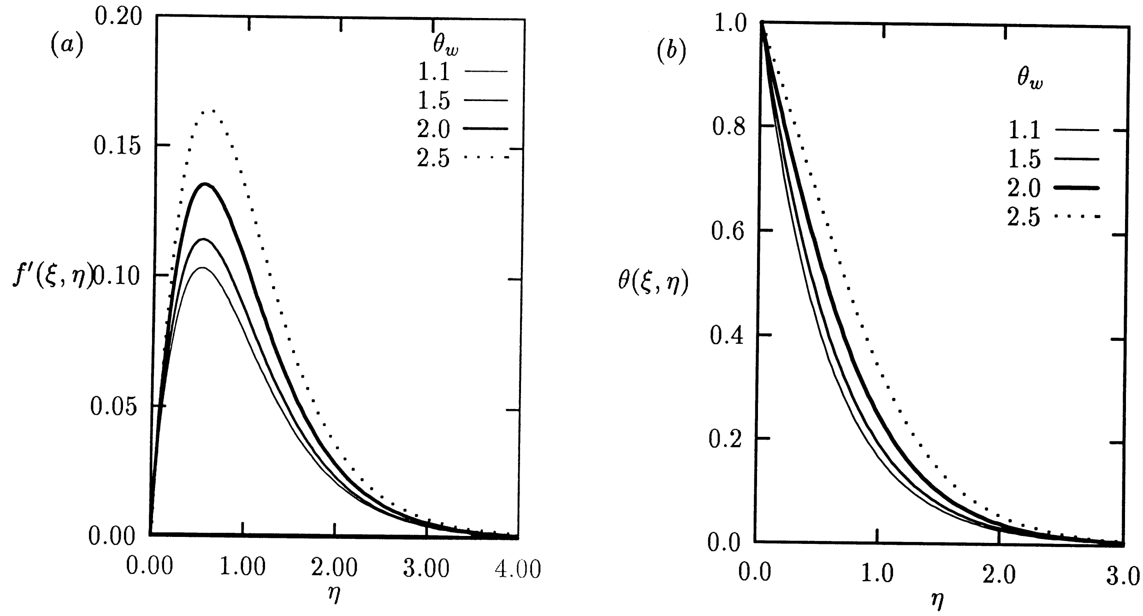

Fig. 8. (a) Velocity and (b) temperature distribution against η for different θ_w with $R_d = 10.0$, $\xi = 2.0$ and $Pr = 1.0$.

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References

[1] V.M. Soundalgekar, H.S. Takhar, N.V. Vighnesam, The combined free and forced convection flow past a semi-infinite vertical plate with variable surface temperature, Nuclear Engineering and Design 110 (1960) 95–98.
[2] A.C. Cogley, W.G. Vincenti, S.E. Giles, Differential approximation for radiative in a non-gray gas near equilibrium, AIAA Journal 6 (1968) 551–553.
[3] M.A. Hossain, H.S. Takhar, Radiation effect on mixed convection along a verticle plate with uniform surface temperature, Heat and Mass Transfer 31 (1996) 243–248.
[4] R. Eichhorn, The effect of mass transfer on free-convection, Journal of Heat Transfer 82 (1960) 260–263.
[5] E.M. Sparrow, R.D. Cess, Free convection with blowing or suction, Journal of Heat Transfer 83 (1961) 387–396.
[6] J.H. Merkin, Free convection with blowing and suction, International Journal of Heat Mass Transfer 15 (1972) 989–999.
[7] J.F. Clarke, Transpiration and natural convection: the vertical flat plate problem, Journal of Fluid Mechanics 57 (1973) 45–61.
[8] H.J. Merkin, The effects of blowing and suction on free convection boundary layers, International Journal of Heat and Mass Transfer 18 (1975) 237–244.
[9] M. Vedhanayagam, R.A. Altenkirch, R. Eichhorn, A transformation of the boundary layer equations for free convection past a vertical flat plate with arbitrary blowing and wall temperature variations, International Journal of Heat and Mass Transfer 23 (1980) 1286–1288.
[10] J.F. Clarke, N. Riley, Natural convection induced in a gas by the presence of a hot porous horizontal surface, Q. J. Mech. Appl. Math. 28 (1975) 373–396.
[11] J.F. Clarke, N. Riley, Free convection and the burning of a horizontal fuel surface, Journal of Fluid Mechanics 74 (1976) 415–431.
[12] H.T. Lin, W.S. Yu, Free convection on a horizontal plate with blowing and suction, Transactions of ASME Journal of Heat Transfer 110 (1988) 793–796.
[13] H.B. Keller, Numerical methods in boundary layer theory, Annual Review of Fluid Mechanics 10 (1978) 417–433.
[14] M.M. Ali, T.S. Chen, B.F. Armaly, Natural convection–radiation interaction in boundary-layer flow over horizontal surfaces, AIAA Journal 22 (1984) 1797–1803.
[15] R. Siegel, J.R. Howell, Thermal Radiation Heat Transfer, McGraw-Hill, New York, 1972.
[16] E.M. Sparrow, H.S. Yu, Local non-similarity thermal boundary layer solutions, Transactions of ASME Journal of Heat Transfer 93 (1971) 328–334.
[17] T.S. Chen, Parabolic systems: local non-similarity method, in: W.J. Minkowycz (Ed.), Handbook of Numerical Heat Transfer, Wiley, New York, 1988. Chap. 5.
[18] M.A. Hossain, N. Banu, A. Nakayama, Non-Darcy forced convection boundary layer flow over a wedge embedded in a saturated porous medium, Numerical Heat Transfer Part A 26 (1994) 399–414.
[19] T. Cebeci, P. Bradshaw, Physical and Computational Aspects of Convective Heat Transfer, Springer, New York, 1984.
[20] M.A. Hossain, Effect of transpiration on combined heat and mass transfer in mixed convection along a vertical plate, International Journal of Energy Research 16 (1992) 761–769.

[21] J.C. Butcher, Implicit Runge–Kutta processes, Math. Comp. 18 (1964) 50–55.

[22] P.R. Nachtsheim, P. Swigert, Satisfaction of asymptotic boundary conditions in numerical solution of systems of non-linear equation of boundary layer type. NASA TN D-3004, 1965.

[23] T.Y. Na, Computational Method in Engineering Boundary Value Problems, Academic Press, New York, 1979.

[24] M.N. Özişik, Radiative Transfer and Interactions with Conduction and Convection, Wiley, New York, 1973.